Contrast Analysis

Statistics for Psychology II

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Outline

- Contrast analysis
- Type of contrast analysis
  - A priori
  - Post hoc
- Special types of contrast analysis

Introduction

- Contrast analysis is to explore and interpret the pattern of differences between groups
  - E.g., compare the life satisfaction between civil servants, employees in private sectors, business owners, and university students
- The omnibus test in ANOVA only tells whether or not the population means are all equal. If not, how do we find out the source of difference?
- In this chapter, specific types of comparisons aiming to answer specific research questions are introduced, such as compare between students and non-students.
  - Combine Groups 1, 2, and 3 as a new group and compare it with Group 4
- If multiple contrasts should be conducted, the familywise error rate must be controlled.

Contrast Analysis

- Compare between four experiment groups to test the effectiveness of blood pressure medicine

New Drug	Original Drug	Placebo	No Drug
μ_1	μ_2	μ_3	μ_4

> The omnibus test in one-way ANOVA shows that the four population means are not all equal. That is, we reject $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$.

- We want to test if there are mean differences between specific groups. Contrast analysis can be used to do that.
  - New drug vs. original drug: $\mu_1 - \mu_2$
  - Drug vs. placebo: $\frac{(\mu_1 + \mu_2)}{2} - \mu_3$

Contrast Analysis

- Two types of contrasts
  - **Pairwise comparison**. The difference is only based on two groups such as new vs. original drug
  - **Complex (or customized) comparison**. The difference is based on more than two groups, such as blood pressure drugs vs. placebo
- We discussed the pairwise comparison in the last chapter. In this chapter, we will formally discuss all types of contrasts.

Contrast Analysis

- The generic formula of contrast

$$\psi = \sum_{j=1}^{k} c_j \mu_j$$

- For example,
  - $\mu_1 - \mu_2 \rightarrow c_1 = 1, c_2 = -1, c_3 = 0, c_4 = 0$
  - $\frac{\mu_1 + \mu_2}{2} - \mu_3 \rightarrow c_1 = \frac{1}{2}, c_2 = \frac{1}{2}, c_3 = -1, c_4 = 0$

Contrast Analysis

- Rule to make contrasts
  - $\sum_{j=1}^{k} c_j = 0$ (Necessary)
  - The sum of c_j in the positive side is 1 and the sum in the negative side is -1 (Not necessary). This will make the resulting contrast (mean difference) on the metric of the original scale.

Contrast	c_1	c_2	c_3	c_4	Sum	Sum of the positives	Sum of the negatives
$\mu_1 - \mu_2$	1	-1	0	0	0	1	-1
$\frac{\mu_1 + \mu_2}{2} - \mu_3$	0.5	0.5	-1	0	0	1	-1

Contrast Analysis

- Previously, the contrast is defined in the population level. The sample definition is as follows:

$$\hat{\psi} = \sum_{j=1}^{k} c_j \bar{Y}_j$$

- Various complex/customized hypotheses can be tested by setting $\psi = 0$.

$$H_0: \psi = 0$$

$$H_0: \mu_1 - \mu_2 = 0 \quad \leftrightarrow \quad H_0: \mu_1 = \mu_2$$

$$H_0: \frac{\mu_1 + \mu_2}{2} - \mu_3 = 0 \quad \leftrightarrow \quad H_0: \frac{\mu_1 + \mu_2}{2} = \mu_3$$

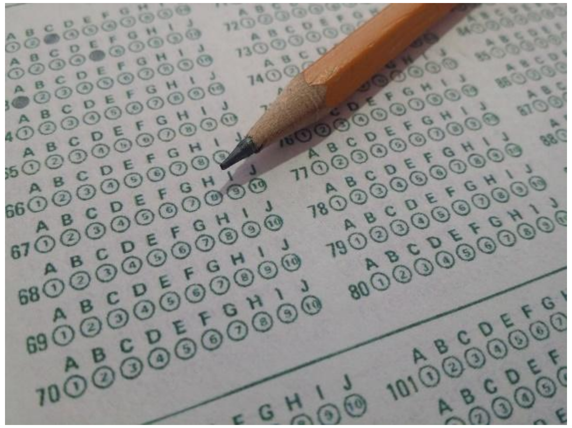
Hypothesis Testing

- Under H_0, $\psi = 0$ in the population. However, the obtained $\hat{\psi}$ is not equal to 0 because of sampling error.
- Hypothesis testing is to check the chance to get $\hat{\psi}$ or more extreme when the null hypothesis is true.
- Thus, we need to find the *p*-value and compare the *p*-value with a pre-determined significance level α (typically .05).

Hypothesis Testing

Compare PISA math test scores between Grade 9 students from 5 schools.

Sample: 50 Grade 9 students are randomly selected in each school.



Schools B and C are private schools.
Schools A, D, and E are public schools.

Research Hypothesis

Public and private schools have different PISA math test scores.

$$H_0: \frac{\mu_A + \mu_D + \mu_E}{3} = \frac{\mu_B + \mu_C}{2}$$

$$H_1: \frac{\mu_A + \mu_D + \mu_E}{3} \neq \frac{\mu_B + \mu_C}{2}$$

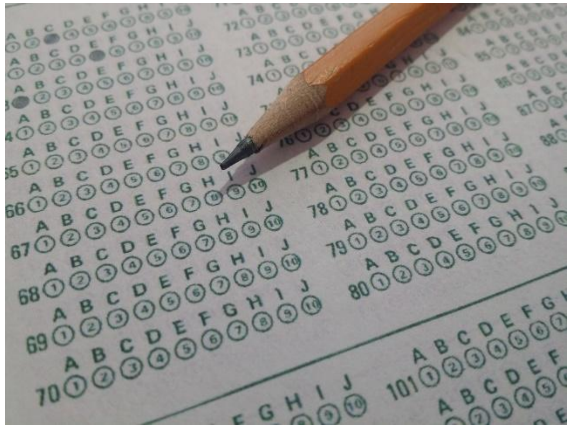
Hypothesis Testing

Compare PISA math test scores between Grade 9 students from 5 schools.
Sample: 50 Grade 9 students are randomly selected in each school.

$$H_0: \frac{\mu_A + \mu_D + \mu_E}{3} = \frac{\mu_B + \mu_C}{2}$$

$$H_0: \psi = \frac{1}{3}\mu_A - \frac{1}{2}\mu_B - \frac{1}{2}\mu_C + \frac{1}{3}\mu_D + \frac{1}{3}\mu_E = 0$$

A	B	C	D	E
Public	Private	Private	Public	Public
0.333	-0.5	-0.5	0.333	0.333



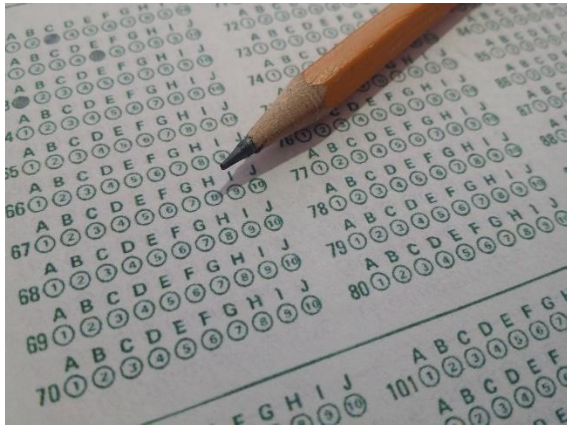
One group includes Schools A, D, and E. The sum of the coefficient must be 1. So, the coefficient of each group is 1/3 or 0.333.

The other group includes Schools B and C. The sum of the coefficient must be -1. So, the coefficient of each group is -1/2 or -0.5.

Hypothesis Testing

Compare PISA math test scores between Grade 9 students from 5 schools.

Sample: 50 Grade 9 students are randomly selected in each school.

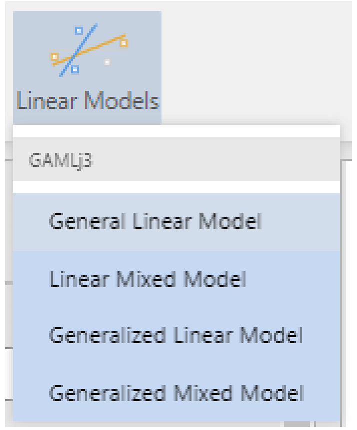


Group Descriptives

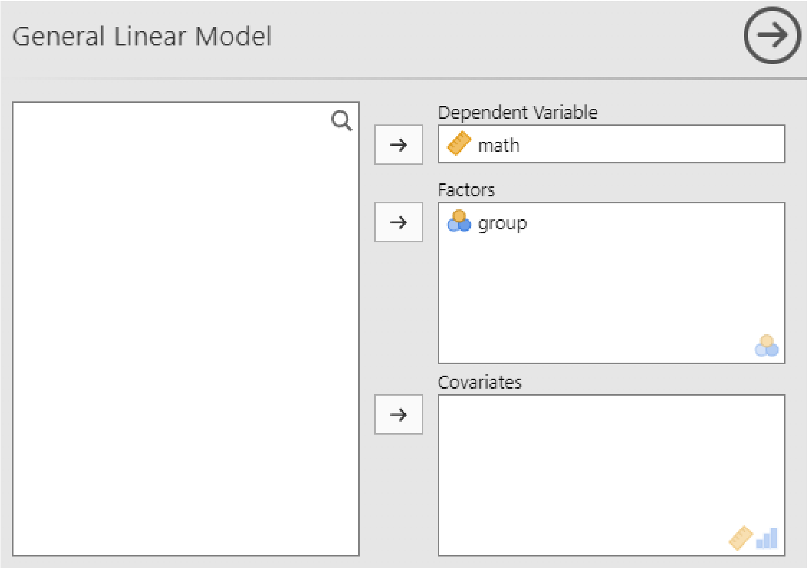
	group	N	Mean	SD	SE
math	A	50	450	73.1	10.33
	B	50	615	114.2	16.15
	C	50	554	95.3	13.48
	D	50	428	21.7	3.07
	E	50	480	46.6	6.59

$$\hat{\psi} = \frac{1}{3}M_A - \frac{1}{2}M_B - \frac{1}{2}M_C + \frac{1}{3}M_D + \frac{1}{3}M_E$$

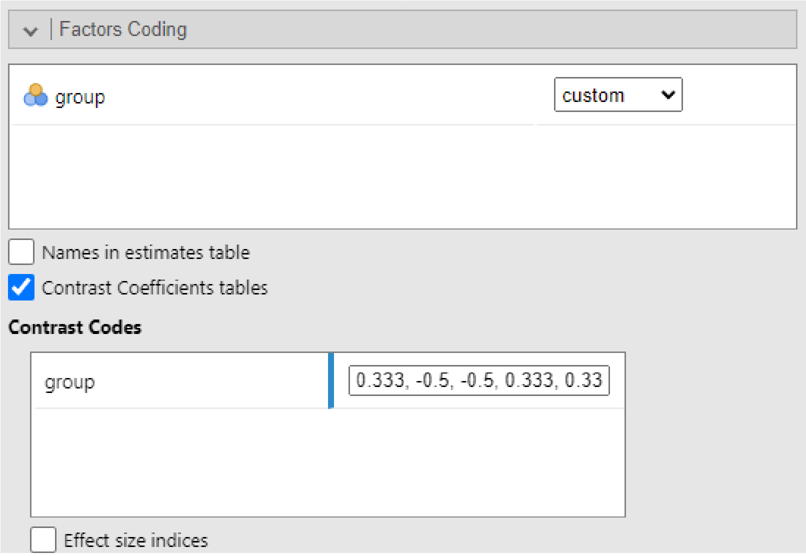
$$= \frac{450}{3} - \frac{615}{2} - \frac{554}{2} + \frac{428}{3} + \frac{480}{3} = -132.04$$



If users wish to specify the contrast coefficients of each group manually,
the `GAMLj3` is needed.



Specify grouping and outcome variables



Go to Factor Coding
Change the drop-down box to custom

Write the contrast coefficients. Separate each group by comma. Fractions are not allowed so the fractions are transformed to decimals.

ANOVA Omnibus tests

	SS	df	F	p	$\eta^2 p$
Model	1208533.056	4	50.173	< .001	0.450
group	1208533.056	4	50.173	< .001	0.450
Residuals	1475337.840	245			
Total	2683870.896	249			

The omnibus test indicated significant differences between schools.

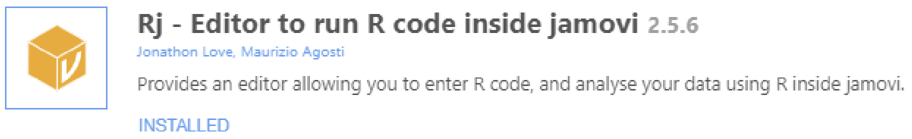
Custom Contrast Tests

				95% Confidence Intervals					
Variable	Contrast	Estimate	SE	Lower	Upper	d	df	t	p
group1	{ 0.333*A, -0.5*B, -0.5*C, 0.333*D, 0.333*E }	−158.515	12.027	−182.204	−134.827	−1.684	245	−13.180	< .001

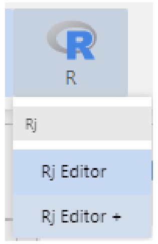
These values were not correct.
The estimate is supposed to be -132.04.

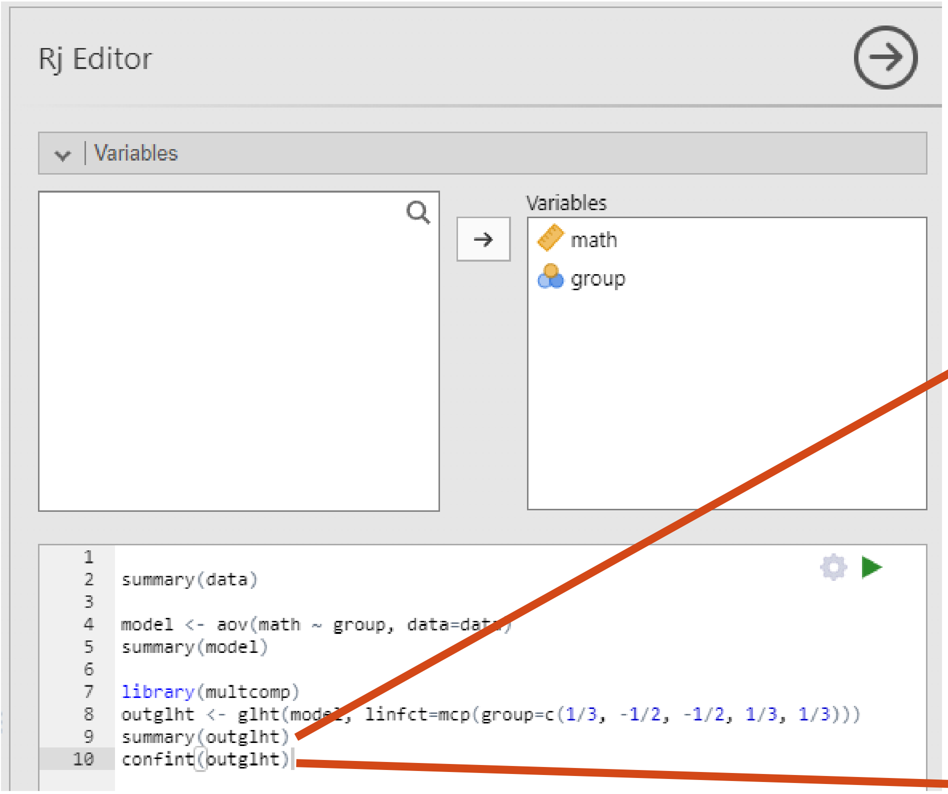
The contrast was significant. The private schools had better math scores than the public schools.

The significant testing was accurate.



Run R in Jamovi using `Rj`. Then, use the `glht` function in the `multcomp` package to run a custom contrast to get accurate estimates and CI.





Rj Editor +

```
 group       math
 A:50   Min.   :221
 B:50   1st Qu.:430
 C:50   Median :480
 D:50   Mean   :505
 E:50   3rd Qu.:570
        Max.   :863


             Df  Sum Sq Mean Sq F value Pr(>F)
group         4 1208533  302133    50.2 <2e-16 ***
Residuals   245 1475338    6022
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


         Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: User-defined Contrasts


Fit: aov(formula = math ~ group, data = data)

Linear Hypotheses:
       Estimate Std. Error t value Pr(>|t|)
1 == 0     -132         10   -13.2   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- single-step method)
```

The contrast estimate is correct. The *t*-value matches the Jamovi output.

```
         Simultaneous Confidence Intervals

Multiple Comparisons of Means: User-defined Contrasts


Fit: aov(formula = math ~ group, data = data)

Quantile = 1.97
95% family-wise confidence level


Linear Hypotheses:
       Estimate lwr      upr
1 == 0 -132.043 -151.776 -112.311
```

Correct CI

Confidence Interval of the Contrast

- CI represents a range of plausible population values of simple or complex contrasts.
- If the CI does not include 0, the contrast is significantly different from 0. That is, the hypothesized population mean difference exists.

Assumption

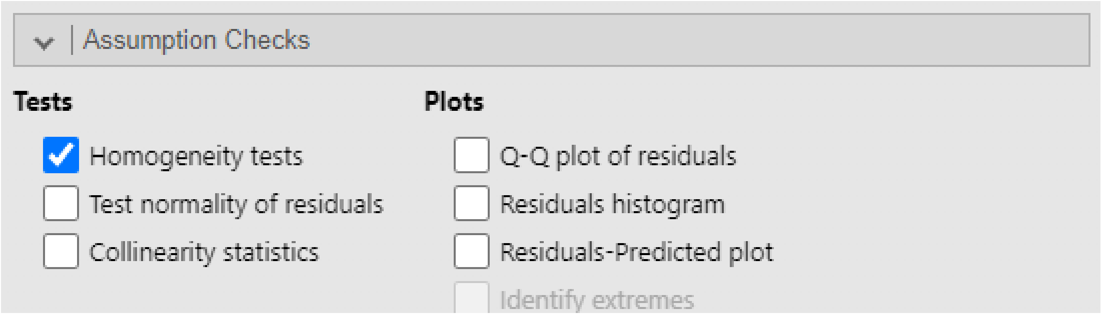
- The assumption of contrast analysis is homogeneity of variance.
  - The robustness of tests in contrast analysis is not as strong as the omnibus test.
  - For example, if Groups 3 and 4 are compared, the pooled *SD* will be calculated from all groups, including Groups 1 and 2 (which are irrelevant in this contrast). If the population variances are not all equal, the results will be biased.
  - Although the group sizes are all equal, the robustness is not better.

Assumption

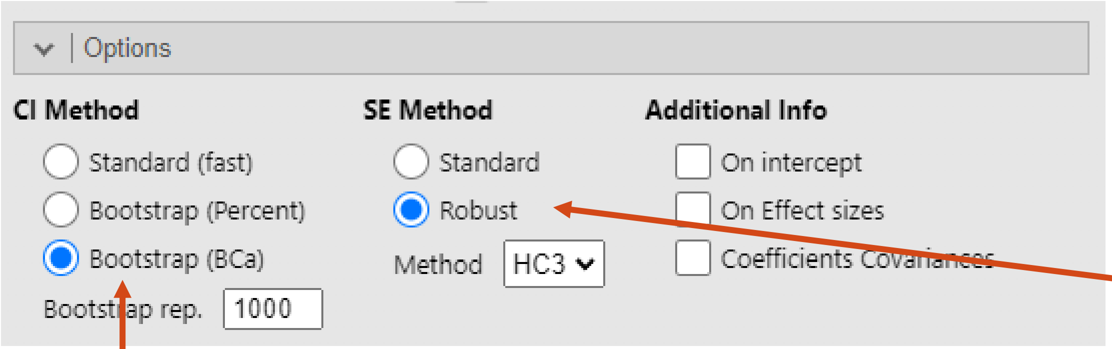
- To check the homogeneity of variance, Levene's test can be used.
- However, when the sample size is small, Levene's test can have lower power to detect the violation.
- Recommendations:
  - Significant result from Levene's test: Use correction
  - Nonsignificant result from Levene's test : Check the ratio of maximum and minimum variances
    - > 3:1, Use alternative methods
    - < 3:1, Standard test can be used

Assumption

- Alternative methods:
  - Pairwise comparison: Independent *t*-test with the Welch correction for the two groups used in comparison
  - Complex comparison: HC3 (the Huber-White correction) and check the *t* and *p*-values.
  - All types of comparison: Bootstrap confidence intervals.



Click homogeneity tests to run Levene's test



Robust standard error can be used in case of heterogeneity of variance. HC3 is the most efficient one.

Users can run the bootstrap confidence interval. However, the obtained CI is not accurate because the contrast estimate is not accurate.

ANOVA Omnibus tests

	SS	df	F	p	$\eta^2 p$
Model	1208533.056	4	50.173	< .001	0.450
group	1208533.056	4	58.101	< .001	0.450
Residuals	1475337.840	245			
Total	2683870.896	249			

Note. Inferential tests and p-values are adjusted for heteroschedasticity.

The result of the omnibus test.

Bootstrap confidence interval is not accurate.

Custom Contrast Tests

				95% Confidence Intervals					
Variable	Contrast	Estimate	SE	Lower	Upper	d	df	t	p
group1	{ 0.333*A, -0.5*B, -0.5*C, 0.333*D, 0.333*E }	−158.515	13.740	−183.916	−133.149	−1.474	245	−11.537	< .001

The robust *t*-test shown significant difference from 0.
This robust test is accurate.

Assumption Checks

Test for Homogeneity of Residual Variance

Test	Statistics	df1	df2	p
Breusch-Pagan Test	36.0	4		< .001
Levene's Test	17.7	4	245	< .001

Note. Levene's test is done only for factors.

Levene's test was significant, indicating that the robust *SE* should be used.

Use R to implement the customized contrast with the HC3 correction..

```r

model <- aov(math ~ group, data=data)
summary(model)

library(multcomp)
outglht <- glht(model, linfct=mcp(group=c(1/3, -1/2, -1/2, 1/3, 1/3)))
summary(outglht)
confint(outglht)

library(sandwich)
library(lmtest)

model0 <- lm(math ~ 1, data=data)
waldtest(model0, model, vcov=vcovHC)

coeftest(model, vcov=vcovHC)

outglht <- glht(model, linfct=mcp(group=c(1/3, -1/2, -1/2, 1/3, 1/3)),
    vcov=vcovHC)
summary(outglht)
confint(outglht)
```

The likelihood ratio test to compare the null model with the model with grouping variables using robust correction.

```
Wald test

Model 1: math ~ 1
Model 2: math ~ group
  Res.Df Df     F Pr(>F)
1    249
2    245  4 58.1 <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The result of robust *t*-test for the regression coefficients.

```
t test of coefficients:

            Estimate Std. Error t value Pr(>|t|)
(Intercept)    449.7       10.4   43.08  < 2e-16 ***
groupB         165.2       19.4    8.53  1.5e-15 ***
groupC         104.5       17.2    6.09  4.4e-09 ***
groupD         -22.2       10.9   -2.04    0.043 *
groupE          30.6       12.4    2.47    0.014 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The robust *t*-test for the customized contrast.

```
         Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: User-defined Contrasts


Fit: aov(formula = math ~ group, data = data)

Linear Hypotheses:
       Estimate Std. Error t value Pr(>|t|)
1 == 0   -132.0       11.4   -11.5   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Adjusted p values reported -- single-step method)
```

The robust CI for the customized contrast.

```
         Simultaneous Confidence Intervals

Multiple Comparisons of Means: User-defined Contrasts


Fit: aov(formula = math ~ group, data = data)

Quantile = 1.97
95% family-wise confidence level


Linear Hypotheses:
       Estimate lwr      upr
1 == 0 -132.043 -154.587 -109.500
```

The (BCa) bootstrap CI for the customized contrast:

```r
library(boot)
ctr <- function(data, indices)
{
  d <- data[indices,] # allows boot to select sample
  model <- lm(math ~ group, data=d)
  outglht <- glht(model, linfct=mcp(group=c(1/3, -1/2, -1/2, 1/3, 1/3)))
  summary(outglht)$test$coefficients
}
results <- boot(data=data, statistic=ctr, R=1000)
boot.ci(results, type="bca")
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 1000 bootstrap replicates

CALL :
boot.ci(boot.out = results, type = "bca")

Intervals :
Level       BCa
95%   (-154.7, -111.3 )
Calculations and Intervals on Original Scale
```

Effect Size

- Standardized mean difference

$$d = \frac{2\hat{\psi}/\sum_{j=1}^{k}|c_j|}{\sqrt{MS_W}}$$

$$d = \frac{\frac{2(-132.04)}{|0.333| + |-0.5| + |-0.5| + |0.333| + |0.333|}}{\sqrt{1475337.84/245}} = \frac{\frac{-264.08}{2}}{\sqrt{6021.79}} = 1.701$$

- Show the difference in the unit of pooled *SD*.
- 0.2 = low, 0.5 = medium, 0.8 = high

Effect Size

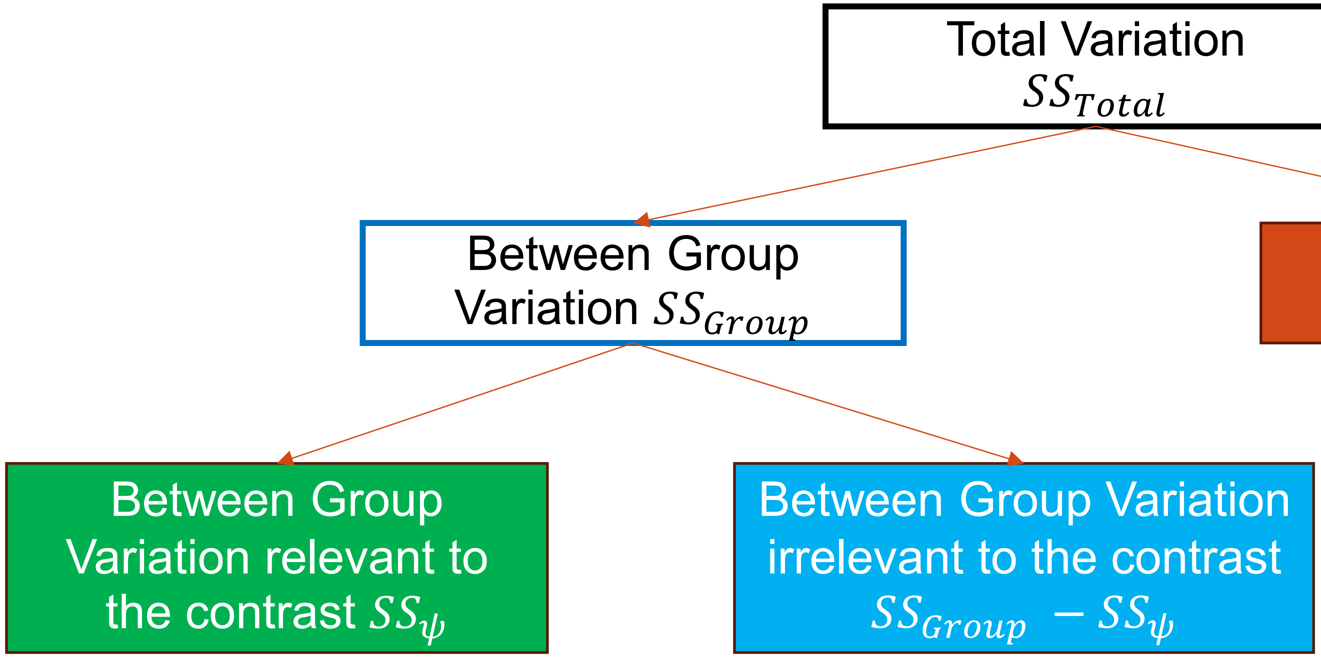
- Because the contrast estimate in Jamovi was not accurate, the obtained Cohen's *d* was not accurate too.

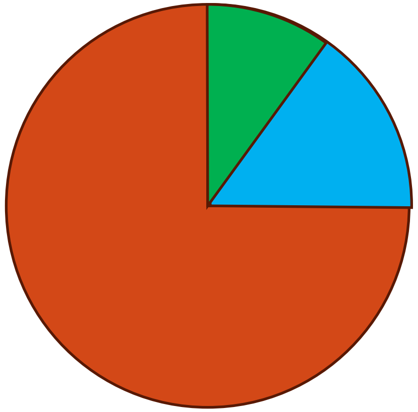
Custom Contrast Tests

Variable	Contrast	Estimate	SE	95% Confidence Intervals Lower	95% Confidence Intervals Upper	d	df	t	p
group1	{ 0.333*A, -0.5*B, -0.5*C, 0.333*D, 0.333*E }	−158.515	13.740	−183.916	−133.149	[illegible]	245	−11.537	< .001

Effect Size

- In general, the proportion of variance explained is the variance of the effect of interest divided by the total variance.
- In ANOVA: $\eta^2 = SS_{Group}/SS_{Total}$, which is the between group variation divided by the total variation.
- However, between group variation should be further divided into the between group variation relevant to the contrast and other between group variation





$$R^2_{\text{Alerting}} = SS_{\psi} / SS_{\text{Group}}$$

$$R^2_{\text{Effect Size}} = SS_{\psi} / SS_{\text{Total}}$$

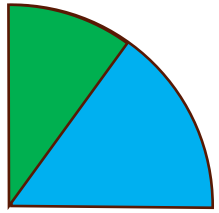
$$R^2_{\text{Contrast}} = SS_{\psi} / (SS_{\psi} + SS_{\text{Error}})$$

$$SS_{\psi} = \frac{\left(\hat{\psi}\right)^2}{\sum_{j=1}^{k} \left(c_j^2 / n_j\right)}$$

Effect Size

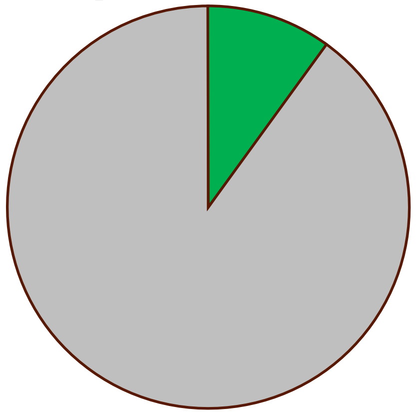
- R^2_{Alerting} shows how much the contrast can explain out of the between group variation. It does not account for the total variation of a variable.

$$R^2_{\text{Alerting}} = SS_\psi / SS_{\text{Group}}$$



- $R^2_{\text{Effect Size}}$ shows how much the contrast can explain out of the total variation of a variable.

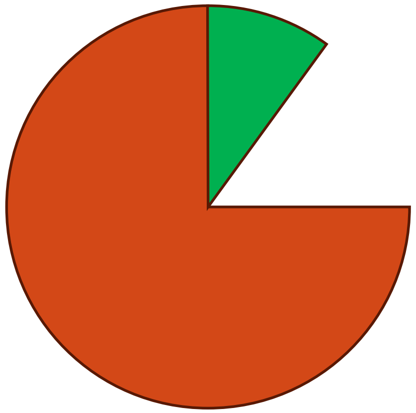
$$R^2_{\text{Effect Size}} = SS_\psi / SS_{\text{Total}}$$



Effect Size

- R^2_{Contrast} shows how much the contrast can explain out of the total variation controlling for the other between group variation.

$$R^2_{\text{Contrast}} = SS_{\psi}/(SS_{\psi} + SS_{\text{Error}})$$



- For example, $\psi = \mu_{\text{New Drug}} - \mu_{\text{No Drug}}$
- Researchers only care about how much the new drug contribute to the symptoms of interest. If the means of old drug or placebo groups change, SS_{Group} and SS_{Total} will change leading to different R^2_{Alerting} and $R^2_{\text{Effect Size}}$. However, SS_{Error} will not change and the R^2_{Contrast} will remain the same.

ANOVA Omnibus tests

	SS	df	F	p	$\eta^2 p$
Model	1208533.056	4	50.173	< .001	0.450
group	1208533.056	4	50.173	< .001	0.450
Residuals	1475337.840	245			
Total	2683870.896	249			

$$SS_{\psi} = \frac{(\hat{\psi})^2}{\sum_{j=1}^{k}\left(c_j^2/n_j\right)} = \frac{(-132.04)^2}{\frac{\left(\frac{1}{3}\right)^2}{50} + \frac{\left(-\frac{1}{2}\right)^2}{50} + \frac{\left(-\frac{1}{2}\right)^2}{50} + \frac{\left(\frac{1}{3}\right)^2}{50} + \frac{\left(\frac{1}{3}\right)^2}{50}} = 1046073.69$$

Total Variation
$SS_{Total} = 2683871$

Between Group Variation
$SS_{Group} = 1208533$

Error Variation
$SS_{Error} = 1475338$

Between Group Variation relevant to the contrast
$SS_{\psi} = 1046074$

Between Group Variation irrelevant to the contrast
$SS_{Group} - SS_{\psi} = 162459.36$

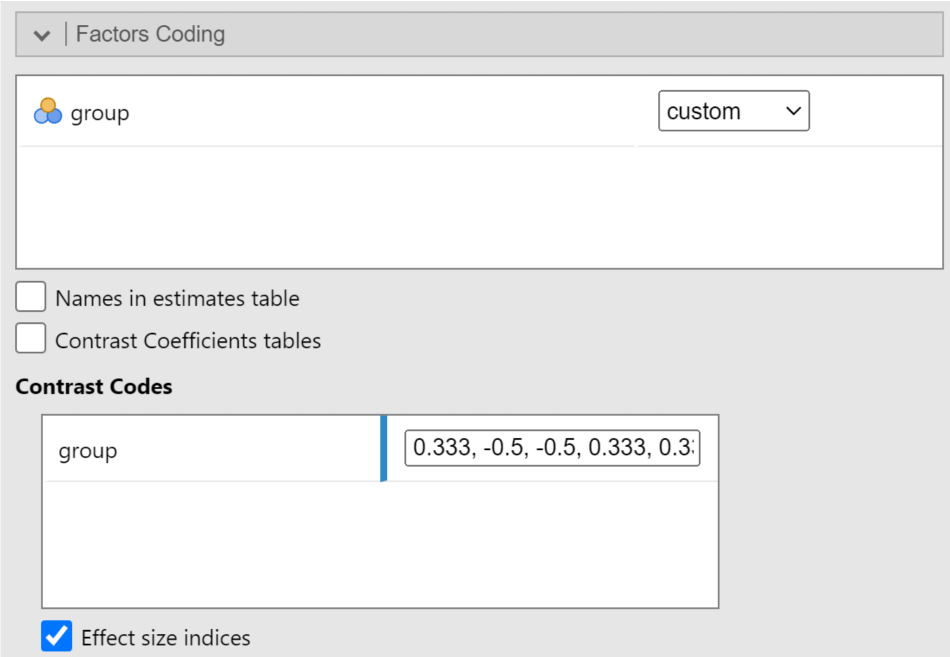
$$R^2_{\text{Alerting}} = \frac{SS_{\psi}}{SS_{\text{Group}}} = \frac{1046074}{1208533} = 87\%$$

$$R^2_{\text{Effect Size}} = \frac{SS_{\psi}}{SS_{\text{Total}}} = \frac{1046074}{2683871} = 39\%$$

$$R^2_{\text{Contrast}} = \frac{SS_{\psi}}{SS_{\psi} + SS_{\text{Error}}} = \frac{1046074}{(1046074 + 1475338)} = 41\%$$

Effect Size

- $R^2_{\text{Alerting}} = 87\%$. The type of schools (either public or private) explains 87% of the between-school variance of math scores.
- $R^2_{\text{Effect Size}} = 39\%$. The type of schools (either public or private) explains 39% of the total variance of math scores.
- $R^2_{\text{Contrast}} = 41\%$. After controlling for the between-school variance of math scores between different public schools and between different private schools, the type of schools (either public or private) explains 41% of the total variance of math scores.



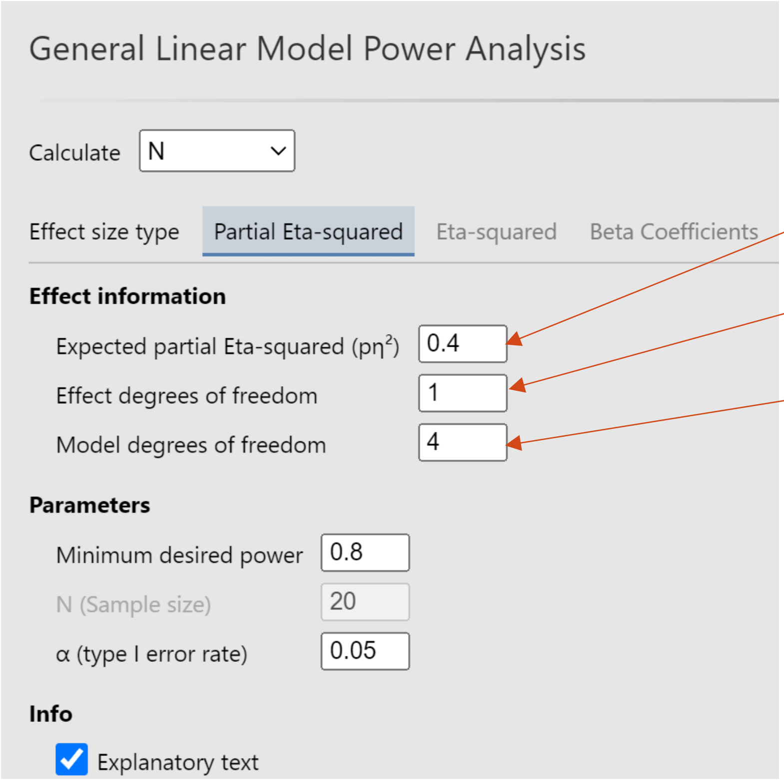
Custom Contrasts effect size indices

Contrast	Statistics	Estimate	95% Confidence Intervals	
			Lower	Upper
{ 0.333*A, -0.5*B, -0.5*C, 0.333*D, 0.333*E }	η^2	2.341	0.000	0.000
	$\eta^2 p$	0.810	0.772	0.839
	ω^2	2.333	0.000	0.000
	$\omega^2 p$	0.807	0.768	0.836
	ε^2	2.338	0.000	0.000
	$\varepsilon^2 p$	0.809	0.771	0.838

As in Jamovi 2.7, the calculated effect size does not match with any of the calculation by hand above. I do not have time to investigate what programs are doing currently. I would recommend users to calculate effect size by hand.

Sample Size Estimation

- The General Linear Model module in the `Pamlj` add-on can be used.



Partial η^2 is conceptually the same as R^2_{Contrast}.

The effect degree of freedom of a contrast is always 1.

The number of groups minus 1.

Sample Size Estimation

General Linear Model

Info

Power parameters are computed for a general linear model with 4 degrees of freedom, for an effect size $p\eta^2 = 0.4$ with 1 degrees of freedom and type I error rate set to 0.05 The model is equivalent to a regression with 4 terms or an ANOVA with 5 groups.

The required Sample size (N) is N=15.

A Priori Power Analysis

N	Effect size	f^2	Power	df	df(res)	α
15	0.400	0.667	0.800	1	10	0.0500

[6]

Power by Effect Size

True effect size	Power to detect	Description
$0 < p\eta^2 \le 0.239$	≤50%	Likely miss
$0.239 < p\eta^2 \le 0.392$	50% – 80%	Good chance of missing
$0.392 < p\eta^2 \le 0.518$	80% – 95%	Probably detect
$p\eta^2 > 0.518$	≥95%	Almost surely detect

Note. Estimated for N=15

The estimated total sample size is 15. That is, three students is needed from each of the 5 schools.

Multiple Contrasts

1 = New Drug / 2 = Old Drug / 3 = Placebo / 4 = No Drug

- From these sets, researchers may want to compare multiple contrasts:
  - Any medication taken vs. no drug: (1, 2, 3) vs. 4
  - Drugs vs. placebo: (1, 2) vs. 3
  - New drug vs. old drug

Multiple Contrasts

- As in any multiple tests, the familywise error rate must be controlled.
- The control method is based on the usage of contrasts:
  - A priori contrast is the contrasts based on research hypotheses stated in advance. The omnibus test in one-way ANOVA is not required.
  - Post hoc comparison is to investigate a difference after the omnibus test indicates differences between groups. No hypotheses about the direction of the group difference is stated.

A Priori Contrasts

- Analysts must have prespecified research hypotheses and transform into contrast coefficients.
- Familywise error rate can be controlled by the Bonferroni, Holm, or Sidak methods. The Holm's method is recommended.

A Priori Contrsts

- From the blood-pressure control medication

Hypothesis	H_0	C_1	C_2	C_3	C_4
Any medication taken vs. no drug	$H_0: \frac{\mu_1 + \mu_2 + \mu_3}{3} = \mu_4$	1/3	1/3	1/3	-1
Drugs vs. placebo	$H_0: \frac{\mu_1 + \mu_2}{2} = \mu_3$	1/2	1/2	-1	0
New drug vs. old drug	$H_0: \mu_1 = \mu_2$	1	-1	0	0

A Priori Contrasts

- Unfortunately, Jamovi 2.7 allows only one custom contrast.
- Analysts can run each contrast separately one at a time. Alternatively, use the preset contrast.
- In this case, this preset contrasts is called "difference"

Contrast Coefficients

Factor: Group

Level=New Drug	Level=Old Drug	Level=Placebo	Level=No Drug	Contrast
−0.500	0.500	0.000	0.000	Old Drug - (New Drug)
−0.333	−0.333	0.667	0.000	Placebo - (New Drug, Old Drug)
−0.250	−0.250	−0.250	0.750	No Drug - (New Drug, Old Drug, Placebo)

A Priori Contrasts

Eighty high-blood-pressure participants were randomly assigned into four groups: (a) new drug, (b) old drug, (c) placebo, and (d) no drug. Seven-day averaged blood pressure were compared between four groups.

Three research hypotheses:

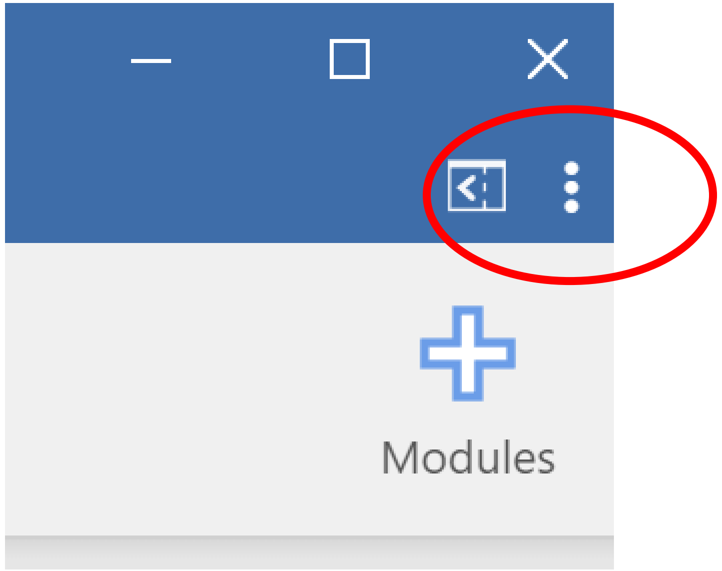
1. The groups taking any medication has lower blood pressure than the group without any drugs.
2. The groups taking active drugs has lower blood pressure than the placebo group.
3. The new-drug group has lower blood pressure than the old-drug group.

$$H_0\colon \frac{\mu_1 + \mu_2 + \mu_3}{3} = \mu_4$$

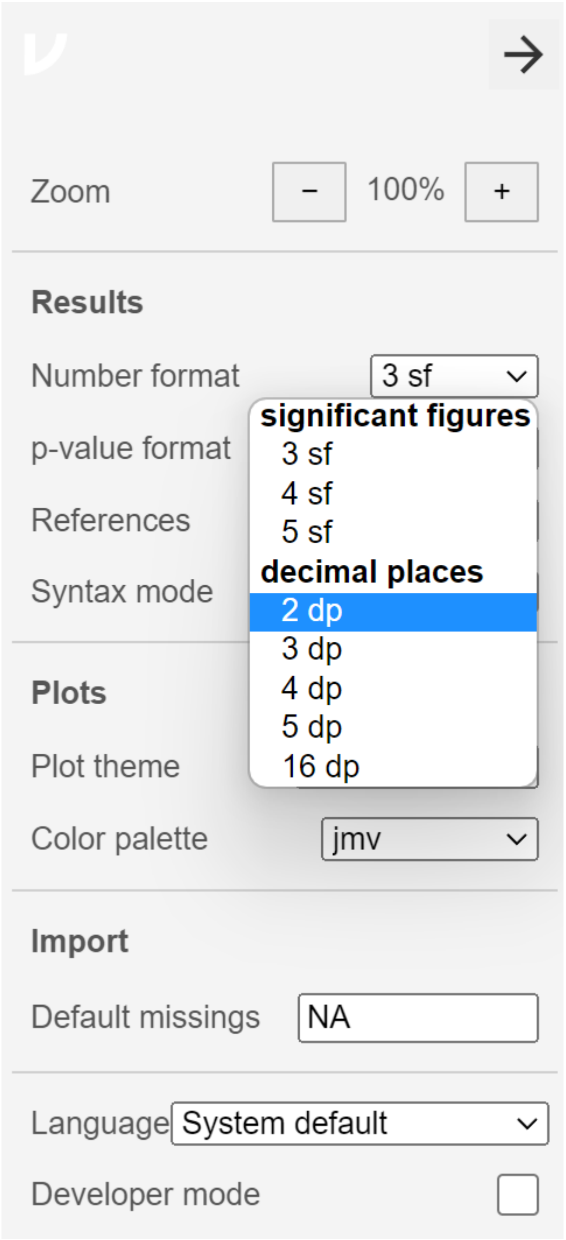
$$H_0\colon \frac{\mu_1 + \mu_2}{2} = \mu_3$$

$$H_0\colon \mu_1 = \mu_2$$

C_1	C_2	C_3	C_4
1/3	1/3	1/3	-1
1/2	1/2	-1	0
1	-1	0	0



Adjust the decimal points in the outputs.



Group Descriptives

	group	N	Mean	SD	SE
bp	New Drug	20	122.80	14.61	3.27
	Old Drug	20	116.30	14.65	3.28
	Placebo	20	136.10	19.44	4.35
	No Drug	20	156.40	15.65	3.50

$$\hat{\psi}_1 = \frac{\mu_1 + \mu_2 + \mu_3}{3} - \mu_4 = \frac{122.8 + 116.3 + 136.1}{3} - 156.4 = -31.33$$

$$\hat{\psi}_2 = \frac{\mu_1 + \mu_2}{2} - \mu_3 = \frac{122.8 + 116.3}{2} - 136.1 = -16.55$$

$$\hat{\psi}_3 = \mu_1 - \mu_2 = 122.8 - 116.3 = 6.5$$

ANOVA Omnibus tests

	SS	df	F	p	η^2p
Mo[illegible]	[illegible]	3	[illegible]	[illegible]	0.485
group	1880[illegible]	[illegible]	[illegible]	< .001	0.485
Res[illegible]	[illegible]	76			
Tota[illegible]	38775.200	79			

Omnibus test is not necessary for a priori contrasts.

Parameter Estimates (Coefficients)

				95% Confidence Intervals					
Names	Effect	Estimate	SE	Lower	Upper	β	df	t	p
(Intercept)	(Intercept)	132.900	1.813	129.290	136.510	−0.000	76	73.324	< .001
group1	Old Drug - (New Drug)	−6.500	5.127	−16.710	3.710	−0.293	76	−1.268	0.209
group2	Placebo - (New Drug, Old Drug)	16.550	4.440	7.708	25.392	0.747	76	3.728	< .001
group3	No Drug - (New Drug, Old Drug, Placebo)	31.333	4.186	22.997	39.670	1.414	76	7.486	< .001

[5]

The familywise error rate need to be controlled externally.

The estimate was correct. CI is also trustworthy.

Contrast Coefficients

Factor: group

Level=New Drug	Level=Old Drug	Level=Placebo	Level=No Drug	Contrast
−0.500	0.500	0.000	0.000	Old Drug - (New Drug)
−0.333	−0.333	0.667	0.000	Placebo - (New Drug, Old Drug)
−0.250	−0.250	−0.250	0.750	No Drug - (New Drug, Old Drug, Placebo)

Robust *SE* can be used here. Also, bootstrap CI can be used too because the contrast estimates were correct.

Check the contrast whether it is what you expected.

A Priori Contrast

- From the research hypotheses:

Use the Holm's method:

Contrasts	*p*	Rank	α	Sig
$H_0: \frac{\mu_1 + \mu_2 + \mu_3}{3} = \mu_4$	< .001	1	.05/3 = .017	YES
$H_0: \frac{\mu_1 + \mu_2}{2} = \mu_3$	< .001	2	.05/2 = .025	YES
$H_0: \mu_1 = \mu_2$	.209	3		NO

A Priori Contrast

- This research study compares the effectiveness of the new blood pressure reduction drug. The random assignment separate 80 participants into four groups: one experiment group (new drug) and three control groups (old drug, placebo, and no drug). Table 1 show the descriptive statistics of blood pressure of each experimental condition.

Table 1 showed the means and standard deviations of each experiment group. The size of each group is 20.

Groups	*M*	*SD*
New Drug	122.80	14.61
Old Drug	116.30	14.65
Placebo	136.10	19.44
No Drug	156.40	15.65

A Priori Contrast

- Three hypotheses were tested using the Holm's familywise error rate correction. The results showed that (a) the three medication groups (new drug, old drug, and placebo) had significant lower blood pressure than the no-drug group, *d* = 31.33, *t*(76) = 7.49, *p* < .001 (less than .05/3 = .167), (b) the two active drug groups (new and old drug) had significant lower means than the placebo group, *d* = 16.55, *t*(76) = 3.73, *p* < .001 (less than .05/2 = .025), (c) the new drug group was not significant different from the old drug group, *d* = 6.50, *t*(76) = -1.27, *p* = .20 (greater than .05). Note that *d* is the raw mean difference.

A Priori Contrast

- A priori contrast can be implemented from 1 contrast to many contrasts. The number of tested contrasts can be over the number of groups.
  - The more the number of contrasts, the more the familywise error rate needs to be controlled
- Sometimes, a priori contrast result was significant but the omnibus test was not significant. Trust the priori contrast when you have a priori research hypothesis.

Post Hoc Comparisons

- A post hoc comparison is conducted when analysts know and get the significant omnibus test.
- Analysts do not have any directions of the mean differences or any target comparisons. They would like to explore the differences after they found significant omnibus test.
  - For example, Thai participants from four universities were significant different in attitude toward luxury cars.

Post Hoc Comparisons

- In a priori hypothesis, analysts would not know the means of each group. They made hypotheses based on their literature or logical thinking.
- If researchers saw the means of each group and they wish to make a hypothesis, they do not make a priori hypothesis. They make hypothesis based on the obtained means. This is obviously post hoc comparisons.

Universities	*M*	*SD*
C	35.15	5.60
T	32.15	4.97
K	24.08	5.36
A	45.70	4.50

They may make the following hypotheses based on the obtained means as follows:

$$H_0: \mu_C = \mu_K$$

$$H_0: \frac{\mu_C + \mu_T}{2} = \mu_K$$

Analysts saw C has the higher mean and K has the lower mean. They intentionally compare these two groups. This is post hoc comparison.

Post Hoc Comparisons

- The post hoc comparison, then, must control the familywise error rate more strictly. Do not reach the conclusion of significant mean differences too easily.
  - If researchers wish to make a pairwise comparison, all possible pairwise comparisons must be accounted for.
  - If researchers wish to make a complex comparison, all possible complex (including pairwise) comparisons must be accounted for.

Post Hoc Comparisons

- Last chapter, pairwise post hoc comparisons has been discussed by listing all possible pairwise comparisons.
  - Tukey's and Games-Howell (or Tukey with HC3) can be used.
- If the post hoc comparisons involve complex comparisons, use
  - The Scheffe's method if homogeneity of variance is assumed.
  - The Brown-Forsythe's method if homogeneity of variance is not assumed. Alternatively, use robust *SE* (HC3).
  - Avoid using these methods if post hoc and complex comparisons are not used because the power is severely compromised.

Post Hoc Comparisons

- The problem of implementing Jamovi and most programs is that they cannot list all possible complex comparisons. They instead show all pairwise comparisons and show the *p*-values based on the Scheffe's method (which is inappropriate).
- The easiest way is to use Scheffe's method (with standard or robust method) is to find the critical values of any contrasts given the Scheffe's method.
  - Critical value = $\pm\sqrt{(k-1), F_{.05,k-1,N-k}}$
  - Then, compare the obtained *t* statistic with this critical value.

Post Hoc Comparisons

A survey explored the attitude toward luxurious cars from undergraduates from four universities (C, T, K, A)

Group Descriptives

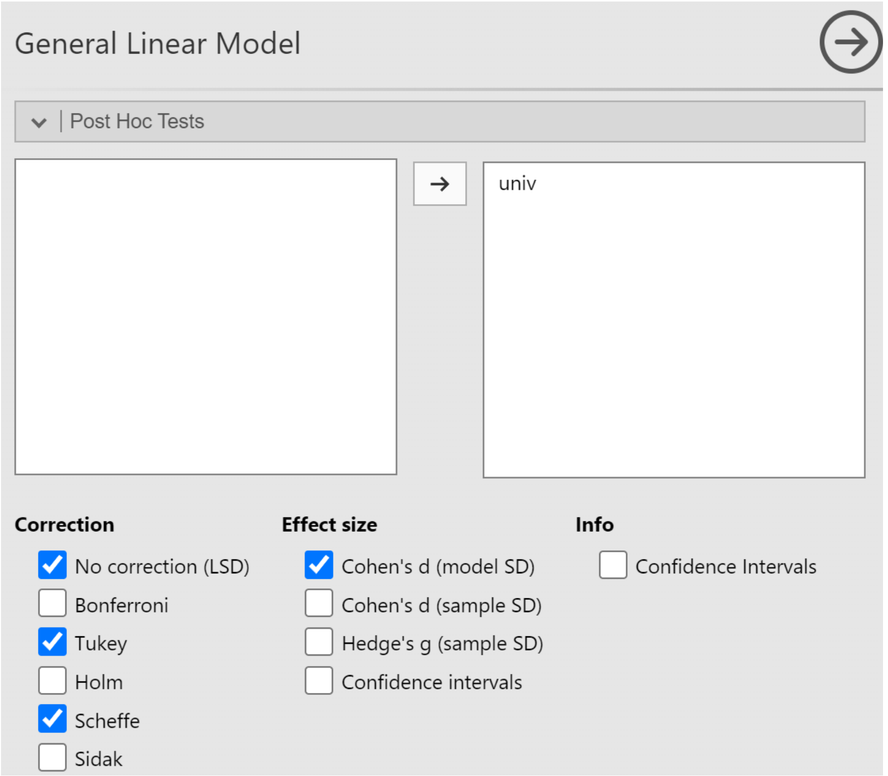
	univ	N	Mean	SD	SE
att	C	72	35.15	5.60	0.66
	T	84	32.15	4.97	0.54
	K	52	24.08	5.36	0.74
	A	37	45.70	4.50	0.74

ANOVA - att

	Sum of Squares	df	Mean Square	F	p
univ	10471.21	3	3490.40	130.06	< .001
Residuals	6467.73	241	26.84		

[3]

Undergraduates from four universities had significant difference in attitude toward luxurious cars.



Implement pairwise comparisons using Tukey's method

Post Hoc comparison: univ

Comparison									
univ	vs	univ	Difference	SE	t	df	p	p_{tukey}	$p_{scheffe}$
C	-	T	3.00	0.86	3.49	241	< .001	0.003	0.008
C	-	K	11.08	1.00	11.05	241	< .001	< .001	< .001
C	-	A	−10.55	1.00	−10.54	241	< .001	< .001	< .001
T	-	K	8.08	0.93	8.70	241	< .001	< .001	< .001
T	-	A	−13.55	0.93	−14.62	241	< .001	< .001	< .001
K	-	A	−21.63	1.06	−20.38	241	< .001	< .001	< .001

Using the Tukey's method, all pairwise comparisons were significantly different.

Find the critical value of the Scheffe's test

Critical value = $\pm\sqrt{(k-1), F_{.05,k-1,N-k}}$

We got k = 4 and N = 245.

Using R to find the critical value.

```
qf(0.95, 4-1, 241)

sqrt((4-1)*qf(0.95, 4-1, 241))
```

R

```
[1] 2.642

[1] 2.815
```

Use the critical value of ± 2.815 to evaluate the comparisons as you want.

$$H_0: \frac{\mu_C + \mu_T}{2} = \frac{\mu_K + \mu_A}{2}$$

Custom Contrast Tests

$|t| < 2.815 \rightarrow ns$

Variable	Contrast	Estimate	SE	95% Confidence Intervals Lower	95% Confidence Intervals Upper	d	df	t	p
univ1	{ 1*C, 1*T, -1*K, -1*A }	−0.62	0.34	−1.30	0.07	−0.23	241	−1.81	0.071

$$H_0: \frac{\mu_C + \mu_K}{2} = \frac{\mu_T + \mu_A}{2}$$

Custom Contrast Tests

$|t| > 2.815 \rightarrow sig$

Variable	Contrast	Estimate	SE	95% Confidence Intervals Lower	95% Confidence Intervals Upper	d	df	t	p
univ1	{ 1*C, -1*T, 1*K, -1*A }	−4.66	0.34	−5.34	−3.97	−1.76	241	−13.64	< .001

$$H_0: \frac{\mu_C + \mu_A}{2} = \frac{\mu_T + \mu_K}{2}$$

Custom Contrast Tests

$|t| > 2.815 \rightarrow sig$

Variable	Contrast	Estimate	SE	95% Confidence Intervals Lower	95% Confidence Intervals Upper	d	df	t	p
univ1	{ 1*C, -1*T, -1*K, 1*A }	6.16	0.34	5.47	6.84	2.32	241	18.03	< .001

$$H_0: \frac{\mu_C + \mu_T + \mu_K}{3} = \mu_A$$

Custom Contrast Tests

$|t| > 2.815 \rightarrow sig$

Variable	Contrast	Estimate	SE	95% Confidence Intervals Lower	95% Confidence Intervals Upper	d	df	t	p
univ1	{ 1*C, 1*T, 1*K, -3*A }	−3.81	0.21	−4.27	−3.35	−2.34	241	−18.14	< .001

Post Hoc Comparisons

- It is quite rare to run post hoc complex comparisons.
- It is possible that the omnibus test found significant differences but all pairwise comparisons were not significant.
- Theoretically, if the omnibus test was significant, at least one complex comparison with the Scheffe's adjustment was significant.

Orthogonal Contrasts

- Two contrasts, ψ_1 and ψ_2, are orthogonal when the contrasts have the following properties:

$$\sum_{j=1}^{k} c_j = 0 \text{ for all contrasts and } \sum_{j=1}^{k} \frac{c_{1j}c_{2j}}{n_j} = 0$$

- If sample size in each group is equal, this equation will be reduced to

$$\sum_{j=1}^{k} c_j = 0 \text{ for all contrasts and } \sum_{j=1}^{k} (c_{1j}c_{2j}) = 0$$

- To claim three or more contrasts are orthogonal, all pairs of contrasts must be orthogonal.

Orthogonal Contrasts

- For example, if sample size is equal,

C_1	C_2	C_3	C_4
1/3	1/3	1/3	-1
1/2	1/2	-1	0
1	-1	0	0

$$\left(\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{1}{3}\times -1\right)+(-1\times 0)=0$$

$$\left(\frac{1}{3}\times 1\right)+\left(\frac{1}{3}\times -1\right)+\left(\frac{1}{3}\times 0\right)+(-1\times 0)=0$$

$$\left(\frac{1}{2}\times 1\right)+\left(\frac{1}{2}\times -1\right)+(-1\times 0)+(0\times 0)=0$$

All contrasts are orthogonal.

Orthogonal Contrasts

- The statistical beauty of orthogonal contrasts is that

$$SS_{Group} = \sum_{j=1}^{k-1} SS_{\psi_j}$$ if all contrasts are orthogonal.

- Another good property is that when a target contrast is considered with or without other orthogonal contrasts, the interpretation of the target contrast does not change.
- The most important thing is that the contrasts must follow the research hypotheses. They do not need to be orthogonal.

Orthogonal Contrasts

- For example, if the sample size is equal, no matter the researcher tests the contrast
  - $H_0\colon \frac{\mu_1+\mu_2+\mu_3}{3} = \mu_4$ alone,
  - $H_0\colon \frac{\mu_1+\mu_2+\mu_3}{3} = \mu_4$ with $H_0\colon \frac{\mu_1+\mu_2}{2} = \mu_3$, or
  - $H_0\colon \frac{\mu_1+\mu_2+\mu_3}{3} = \mu_4$ with $H_0\colon \frac{\mu_1+\mu_2}{2} = \mu_3$ and $H_0\colon \mu_1 = \mu_2$,
- The estimate of $\psi = \frac{1}{3}\mu_1 + \frac{1}{3}\mu_2 + \frac{1}{3}\mu_3 - \mu_4$ in all tests remains the same.

Orthogonal Contrasts

- However, if sample size is equal, when testing the following contrasts
  - $H_0: \mu_1 = \mu_2$ alone. The estimate represent the difference between Groups 1 and 2.
  - $H_0: \mu_1 = \mu_2, H_0: \mu_1 = \mu_3, H_0: \mu_1 = \mu_4$ all together. The estimate of the first contrast, instead, indicates the difference between Group 2 and the grand mean $\left(\frac{\mu_1+\mu_2+\mu_3+\mu_4}{4}\right)$
- The reason is that $H_0: \mu_1 = \mu_2, H_0: \mu_1 = \mu_3, H_0: \mu_1 = \mu_4$ are not orthogonal.

Standard Sets of Contrasts

- **Dummy**:
  - Each contrast has focal group (coded as 1). The remaining groups are coded as 0.
  - The baseline group is the group that has not been used as focal group.
  - It does not follow contrasts rule because $\sum_{j=1}^{k} c_j \neq 0$.
  - All three contrasts must be put simultaneously so the estimates are interpreted as the difference between focal group and the baseline group.

Factor: univ

Level=C	Level=T	Level=K	Level=A	Contrast
0.00	1.00	0.00	0.00	T - C
0.00	0.00	1.00	0.00	K - C
0.00	0.00	0.00	1.00	A - C

University T – University C
University K – University C
University A – University C

Standard Sets of Contrasts

- **Simple**:
  - Each contrast has focal group (coded as (*k*-1)/*k*). The remaining groups are coded as -1/*k*.
  - The baseline group is the group that has not been used as focal group.
  - This is simply the dummy coding but adjust to get $\sum_{j=1}^{k} c_j = 0$.
  - All three contrasts are not orthogonal.
  - If considered alone, the estimate is the difference between focal group and the average of the remaining groups.
  - If put simultaneously, the resulting estimates are the focal group and the baseline group

Factor: univ

Level=C	Level=T	Level=K	Level=A	Contrast	
−0.25	0.75	−0.25	−0.25	T - C	University T – University C
−0.25	−0.25	0.75	−0.25	K - C	University K – University C
−0.25	−0.25	−0.25	0.75	A - C	University A – University C

Standard Sets of Contrasts

- **Difference**:
  - Contrast 1: Group 2 – Group 1
  - Contrast 2: Group 3 - The average of Group 1 and 2
  - Contrast 3: Group 4 - The average of Group 1, 2, and 3
  - …
  - Contrast *k* - 1: Group *k* - The average of Group 1, 2, …, and *k* - 1
  - If group size is equal, all contrasts are orthogonal. Each can be interpreted as the specified difference.

Factor: univ

Level=C	Level=T	Level=K	Level=A	Contrast	
−0.50	0.50	0.00	0.00	T - C	University T – University C
−0.33	−0.33	0.67	0.00	K - (C, T)	University K - University T, C
−0.25	−0.25	−0.25	0.75	A - (C, T, K)	University A – University C, T, K

Standard Sets of Contrasts

- **Repeated**:
  - Contrast 1: Group 1 – The average of Group 2, 3, … , and *k*
  - Contrast 2: The average of Group 1, 2 – The average of Group 3, 4, … , and *k*
  - Contrast 3: The average of Group 1, 2, 3 – The average of Group 4, 5, … , and *k*
  - …
  - Contrast k-1: The average of Group 1, 2, 3, … , *k* - 1 – Group *k*
  - These contrasts are not orthogonal. When considered simultaneously, each estimate will be the difference between the transition group.
  - Use in longitudinal design or when groups have meaningful order.

Factor: univ

Level=C	Level=T	Level=K	Level=A	Contrast	
0.75	−0.25	−0.25	−0.25	C - T	University C – University T
0.50	0.50	−0.50	−0.50	T - K	University T – University K
0.25	0.25	0.25	−0.75	K - A	University K – University A

Trend Analysis

- Sometimes, the group represents a level of some variables.
- For example, testing the effect of meditation on GPA. Five groups are separated by the daily meditation minutes: 0, 5, 10, 15, and 20 minutes/day.
- Analysts wish to check whether meditation affects GPA. Is the effect in the linear or curvilinear effect.
- The analysis of changes based on the level of groups is referred to as trend analysis.

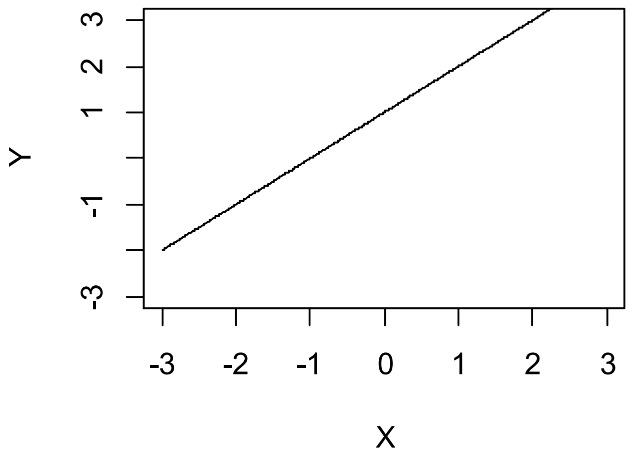
Trend Analysis

- Trend analysis means to set contrast coefficients appropriately so that the resulting contrast estimates mean the linear change, quadratic change, cubic change, …
- In this class, only polynomial change is considered in trend analysis, which is defined as

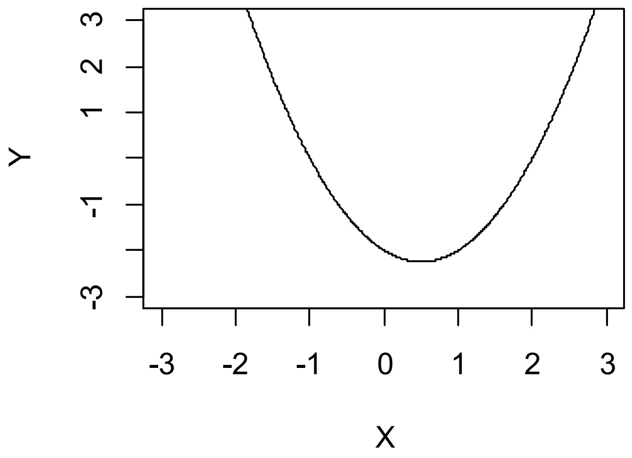
$$Y = \beta_0 + \beta_1 X_j + \beta_2 X_j^2 + \cdots + \beta_{k-1} X_j^{k-1} + \varepsilon_{ij}$$

  - Y = outcome
  - X = the level specified in each group

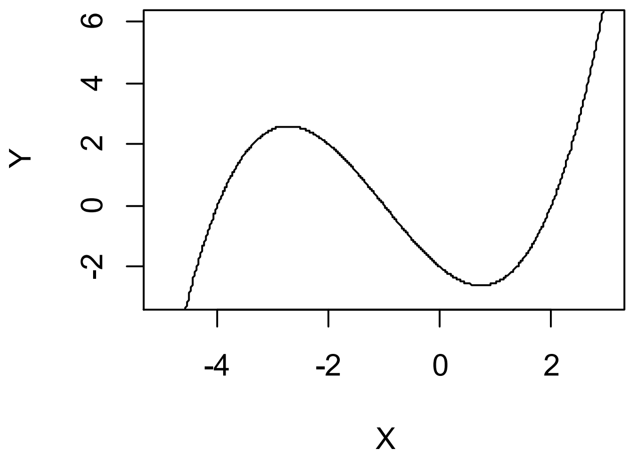
First level polynomial (Linear)



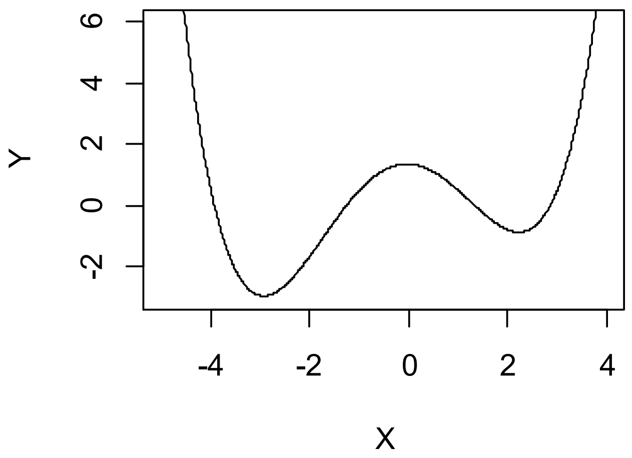
Second Level Polynomial (Quadratic)



Third level polynomial (Cubic)



Fourth Level Polynomial (Quartic)



Trend Analysis

- If the interval of levels are equal (e.g., increased by 5 minutes) and group size is equal, the polynomial contrasts can be used in Jamovi.
- For example, if seven groups have meditation minutes per day as 0, 5, 10, 15, 20, 25, 30. The linear contrasts are as follows:
  - $X_j = \{0,5,10,15,20,25\}$
  - $c_j = X_j - \bar{X}_j = \{-15, -10, -5, 0, 5, 10, 15\}$
- The contrasts can be multiplied by a constant for scaling. The resulting test statistics and significance level remain the same.

Trend Analysis

- Use the following table to specify the contrast for polynomial trend analysis
- See full table in Table 8 of the Appendix in Maxwell et al. (2018)
- `contr.poly()` function in R can calculate the coefficients

The coefficients used for calculating sums of squares are:

Number of treatment	Degree of polynomial	T1	T2	T3	T4	T5	T6	Divisor $k=\sum c_i^2$
		Treatment totals						
2	1	-1	+1					2
3	1	-1	0	+1				2
	2	+1	-2	+1				6
4	1	-3	-1	+1	+3			20
	2	+1	-1	-1	+1			4
	3	-1	+3	-3	+1			20
5	1	-2	-1	0	+1	+2		10
	2	+2	-1	-2	-1	+2		14
	3	-1	+2	0	-2	+1		10
	4	+1	-4	+6	-4	+1		70
6	1	-5	-3	-1	+1	+3	+5	70
	2	+5	-1	-4	-4	-1	+5	84
	3	-5	+7	+4	-4	-7	+5	180
	4	+1	-3	+2	+2	-3	+1	28
	5	-1	+5	-10	+10	-5	+1	252

Trend Analysis

An experiment investigates the performance ratings given to a confederate based on how participants know the confederate rates their attractiveness (from 1 being very unattractive to 5 being very attractive). Participants are randomly assigned to 5 groups (attractiveness ratings = 1, 2, 3, 4, or 5).

Research Hypothesis
The more attractiveness rated, the more performance rating given.

Analysts are also interested in the pattern of changes by using polynomial models.

$$X_j = \{1,2,3,4,5\}$$

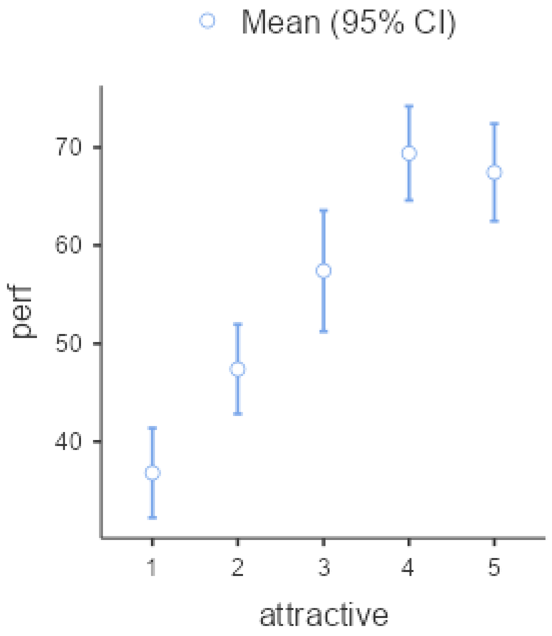
$$c_{\text{Linear}} = \{-2,-1,0,1,2\}$$

$$c_{\text{Quadratic}} = c^2_{\text{Linear}} - \frac{1}{k}\sum c^2_{\text{Linear}} = \{2,-1,-2,-1,2\}$$

5	1	-2	-1	0	+1	+2	10
	2	+2	-1	-2	-1	+2	14
	3	-1	+2	0	-2	+1	10
	4	+1	-4	+6	-4	+1	70

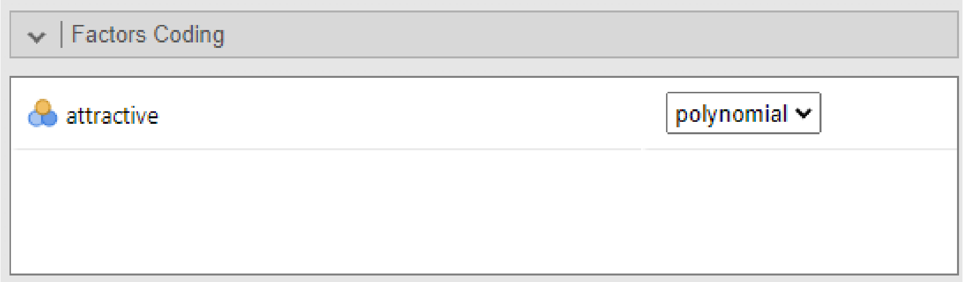
Group Descriptives

	attractive	N	Mean	SD	SE
perf	1	20	36.80	9.78	2.19
	2	20	47.40	9.77	2.19
	3	20	57.40	13.14	2.94
	4	20	69.40	10.27	2.30
	5	20	67.45	10.63	2.38



The more attractive rating received the more performance rating given.

The change is linear at first but when the attractive is over 4, the performance rating is not getting better. The change seems to be curvilinear.



Factor: attractive

Level=1	Level=2	Level=3	Level=4	Level=5	Contrast
−0.63	−0.32	−0.00	0.32	0.63	linear
0.53	−0.27	−0.53	−0.27	0.53	quadratic
−0.32	0.63	0.00	−0.63	0.32	cubic
0.12	−0.48	0.72	−0.48	0.12	quartic

The coefficients are simply the same as the table. The only change is that they were multiplied by a constant to make contrast coefficients sum on both negative and positive sides equal to 1. However, the resulting contrast coefficients do not make sense.

Parameter Estimates (Coefficients)

				95% Confidence Intervals					
Names	Effect	Estimate	SE	Lower	Upper	β	df	t	p
(Intercept)	(Intercept)	55.69	1.08	53.55	57.83	−0.00	95	51.60	< .001
attractive1	linear	26.34	2.41	21.55	31.13	1.62	95	10.92	< .001
attractive2	quadratic	−6.17	2.41	−10.96	−1.38	−0.38	95	−2.56	0.012
attractive3	cubic	−4.22	2.41	−9.01	0.57	−0.26	95	−1.75	0.083
attractive4	quartic	−2.22	2.41	−7.01	2.57	−0.14	95	−0.92	0.361

The highest polynomial power is 4 when the number of group is 5.

The cubic and quartic changes were not significant. The quadratic and linear changes were significant.

Trend Analysis

- When group sizes are equal, the linear, quadratic, and cubic contrasts represent the averaged (linear, quadratic, cubic) changes across all X values. If the linear contrast is significant, it indicates that the average linear change across all X values is positive.
- When group sizes are unequal, the contrasts are not orthogonal. The linear contrast coefficient represents the instantaneous rate of change at $X = \bar{X}$. It is recommended to include only the linear contrast initially to examine the overall linear trend and then add higher-order contrasts one by one to evaluate additional polynomial effects (e.g., quadratic or cubic changes).
- This approach ensures a systematic assessment of both linear and non-linear patterns in the data.

Trend Analysis

- A one-way ANOVA with polynomial contrasts was conducted to examine the effect of confederates' attractiveness ratings (1 = very unattractive, 2, 3, 4, 5 = very attractive) on performance ratings (DV) provided by participants, ranging from 1 to 100. Descriptive statistics for the performance ratings across the five levels of attractiveness are presented in Table 1. As shown in Figure 1, the means suggest a positive trend, with higher levels of attractiveness generally associated with higher performance ratings.

Table 1 shows the means and standard deviations of performance ratings for each confederate attractiveness rating group. The sample size for each condition is 20.

Attractiveness	*M*	*SD*
1 (Very unattractive)	36.80	9.78
2	47.40	9.77
3	57.40	13.14
4	69.40	10.27
5 (Very attractive)	67.45	10.63

Trend Analysis

- The polynomial contrast analysis revealed a significant linear effect of attractiveness on performance ratings, $\hat{\psi} = 1.62, t(95) = -10.92, p < .001$, indicating that as confederates' attractiveness increased, participants' performance ratings also increased linearly. The quadratic effect was also significant, $\hat{\psi} = -0.38, t(95) - 2.56, p = .012$, suggesting a slight curve in the trend. Note that $\hat{\psi}$ is the contrast estimate.
- Higher-order effects, including cubic $\hat{\psi} = -0.26, t(95) = -1.75, p = .083$, and quartic $\hat{\psi} = -0.14, t(95) = -0.92, p = .36$, were not significant. These results suggest that the primary pattern in the data is driven by a strong linear relationship, with some contribution from a quadratic curvature, while higher-order trends are negligible.

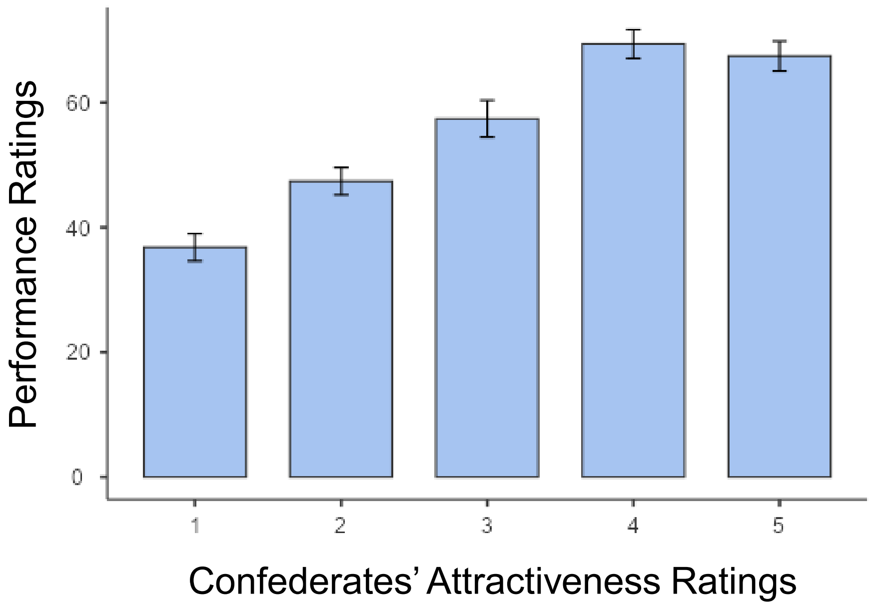


Figure 1 shows the bar chart of the average performance ratings for each confederate attractiveness rating group. The bars represent the standard error of the group means.

Trend Analysis

- Overall, the findings indicate that attractiveness ratings significantly influence performance ratings, with participants rating more attractive confederates higher in performance. The significant quadratic term suggests some deviation from perfect linearity in the pattern, potentially reflecting a diminishing return in ratings for the highest levels of attractiveness.

Trend Analysis

- Trend analysis contrasts do not require the omnibus test.
- If *X* has unequal interval, use `contr.poly()` function in R to find the orthogonal polynomial contrast coefficients. For example, mediation minutes are 0, 5, 10, 20, 40

The number of groups

X values

Use `round()` function for rounding.

```
> round(contr.poly(5, scores=c(0, 5, 10, 20, 40)), 4)
          .L      .Q      .C      ^4
[1,] -0.4743  0.5460 -0.4700  0.2367
[2,] -0.3162  0.0352  0.4223 -0.7214
[3,] -0.1581 -0.3346  0.5143  0.6312
[4,]  0.1581 -0.6516 -0.5705 -0.1578
[5,]  0.7906  0.4051  0.1039  0.0113
```

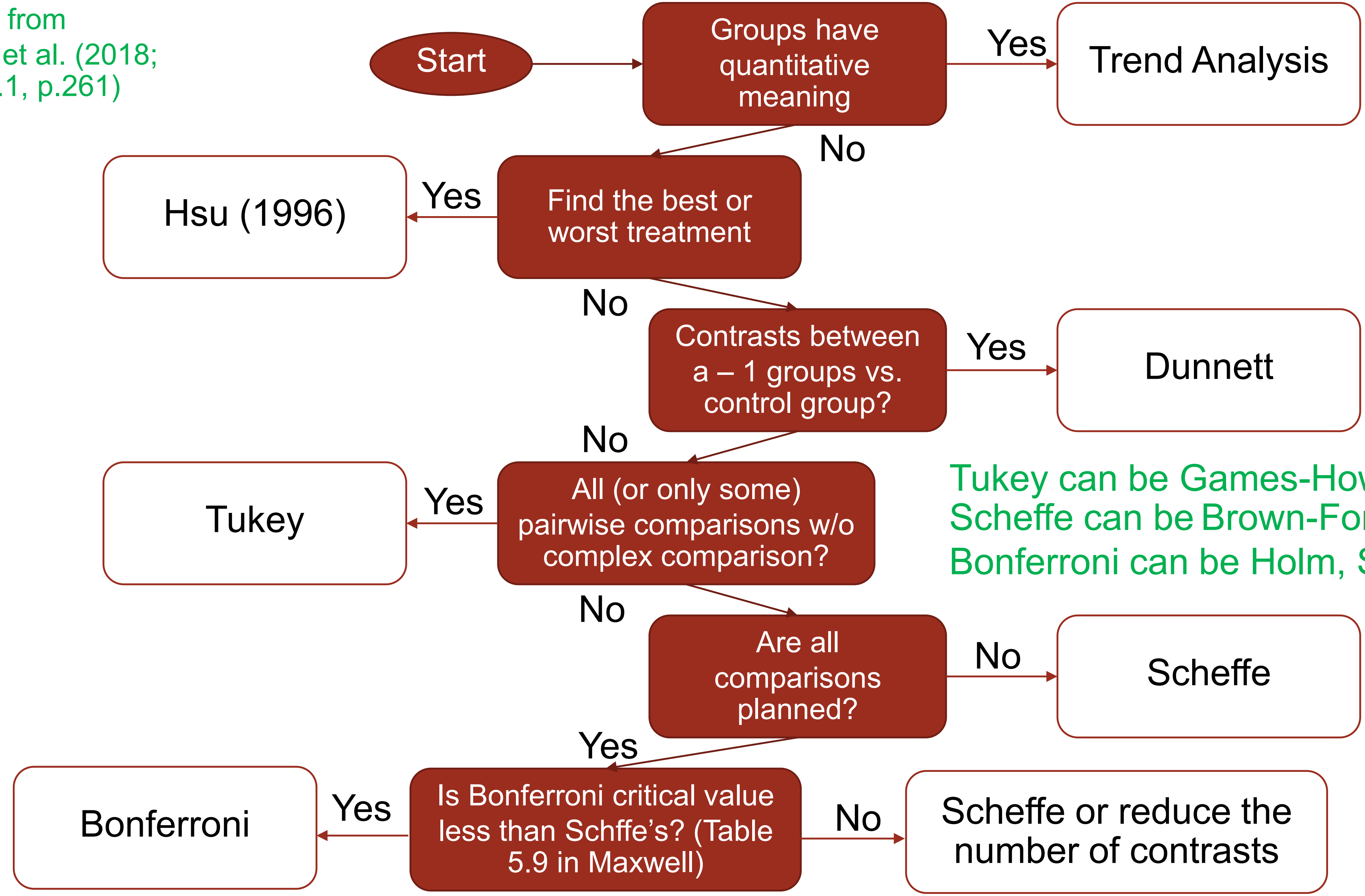
Trend Analysis

- As in other contrast analysis, the homogeneity of variance is required. If it is not assumed, HC3 or bootstrap is needed.

Other Interesting Contrast Analysis

- Dunnett test checks whether the control group is significant different from each of the remaining groups.
- Hsu's (1996) method will systematically check whether the maximum (or minimum) group is truly the maximum (or minimum) group in the population.
  - It is helpful for identifying the best or worst treatment options.

Modified from Maxwell et al. (2018; Figure 5.1, p.261)



Tukey can be Games-Howell
Scheffe can be Brown-Forsythe
Bonferroni can be Holm, Sidak